Sequence of Geometry Modules Aligned with the Standards

Module 1: Congruence, Proof, and Constructions

Module 2: Similarity, Proof, and Trigonometry

Module 3: Extending to Three Dimensions

Module 4: Connecting Algebra and Geometry through Coordinates

Module 5: Circles with and Without Coordinates

Summary of Year

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Geometry

- Triangle congruence and similarity criteria.
- Using coordinates to establish geometric results.
- Calculating length and angle measures.
- Using geometric representations as a modeling tool.
- Using construction tools, physical and computational to draft models of geometric phenomenon.

CCLS Major Emphasis Clusters
Congruence
• Understand congruence in terms of rigid motions
• Prove geometric theorems
Similarity, Right Triangles, and Trigonometry
• Understand similarity in terms of similarity transformations
• Prove theorems using similarity
• Define trigonometric ratios and solve problems involving right triangles
Expressing Geometric Properties with Equations
• Use coordinates to prove simple geometric theorems algebraically
Modeling with Geometry
• Apply geometric concepts in modeling situations

Rationale for Module Sequence in Geometry

Module 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions—translations, reflections, and rotations—and have strategically applied a rigid motion to informally show that two triangles are congruent. In this module, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They build upon this familiar foundation of triangle congruence to develop formal proof techniques. Students make conjectures and construct viable arguments to prove theorems— using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They construct figures by manipulating appropriate geometric tools (compass, ruler, protractor, etc.) and justify why their written instructions produce the desired figure.

Module 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, make sense of and persevere in solving similarity problems, and apply similarity to right triangles to prove the Pythagorean Theorem. Students attend to precision in showing that trigonometric ratios are well defined, and apply trigonometric ratios to find missing measures of general (not necessarily right) triangles. Students model and make sense out of indirect measurement problems and geometry problems that involve ratios or rates.

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Module 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. They reason abstractly and quantitatively to model problems using volume formulas.

Module 4: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students analyze geometric relationships in the context of a rectangular coordinate system, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, relating back to work done in the first module. Students attend to precision as they connect the geometric and algebraic definitions of parabola. They solve design problems by representing figures in the coordinate plane, and in doing so, they leverage their knowledge from synthetic geometry by combining it with the solving power of algebra inherent in analytic geometry.

Module 5: In this module, students prove and apply basic theorems about circles, such as: a tangent line is perpendicular to a radius theorem, the inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students explain the correspondence between the definition of a circle and the equation of a circle written in terms of the distance formula, its radius, and coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane and apply techniques for solving quadratic equations. Students visualize, with the aid of appropriate software tools, changes to a three-dimensional model by exploring the consequences of varying parameters in the model.

Alignment Chart

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
Module 1: **Congruence, Proof, and Constructions** (45 days)	**Experiment with transformations in the plane** **G-CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **G-CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **G-CO.3**[42] Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **G-CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G-CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **Understand congruence in terms of rigid motions** **G-CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **G-CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

[42]Trapezoid is defined as "A quadrilateral with at least one pair of parallel sides" (in preparation for Regents Exams).

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
	G-CO.8　Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **Prove geometric theorems** **G-CO.9**　Prove theorems about lines and angles. *Theorems include[43]: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **G-CO.10**　Prove theorems about triangles. *Theorems include[44]: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **G-CO.11**　Prove theorems about parallelograms. *Theorems include[45]: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*. **Make geometric constructions** **G-CO.12**[46]　Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **G-CO.13**　Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

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[43] Theorems include but are not limited to the listed theorems. Example: Theorems that involve complementary or supplementary angles (in preparation for Regents Exams).

[44] Theorems include but are not limited to the listed theorems. Example: An exterior angle of a triangle is equal to the sum of the two interior opposite angles of the triangle (in preparation for Regents Exams).

[45] Theorems include but are not limited to the listed theorems. Example: A rhombus is a parallelogram with perpendicular diagonals (in preparation for Regents Exams).

[46] The constructions include, but are not limited to, the listed constructions (in preparation for Regents Exams).

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules

Module 2:
 	
 Similarity, Proof, and Trigonometry
 (45 days)
	Understand similarity in terms of similarity transformations
	
	 G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:
	
	      a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
	
	      b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
	
	 G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
	
	 G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
	
	 Prove theorems involving similarity
	
	 G-SRT.4 Prove theorems about triangles. *Theorems include[47]: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
	
	 G-SRT.5 Use congruence and similarity criteria[48] for triangles to solve problems and to prove relationships in geometric figures.
	
	 Define trigonometric ratios and solve problems involving right triangles
	
	 G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

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[47] Theorems include, but are not limited to, the listed theorems. Example: the length of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the lengths of the two segments of the hypotenuse (in preparation for Regents Exams).

[48] ASA, SAS, SSS, AAS, and Hypotenuse-Leg theorems are valid criteria for triangle congruence. AA, SAS, and SSS are valid criteria for triangle similarity (in preparation for Regents Exams).

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
	G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. **G-SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ **Apply geometric concepts in modeling situations** **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★ **G-MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★ **G-MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★
Module 3: **Extending to Three Dimensions** (10 days)	**Explain volume formulas and use them to solve problems**[49] **G-GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ **Visualize relationships between two-dimensional and three-dimensional objects** **G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **Apply geometric concepts in modeling situations** **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a

[49] The (+) standard on the volume of the sphere is an extension of G-GMD.1. It is explained by the teacher in this grade and used by students in G-GMD.3. Note: Students are not assessed on proving the volume of a sphere formula until Precalculus.

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
	tree trunk or a human torso as a cylinder).★
Module 4: **Connecting Algebra and Geometry through Coordinates** (25 days)	**Use coordinates to prove simple geometric theorems algebraically** **G-GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **G-GPE.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **G-GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **G-GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★
Module 5: **Circles with and Without Coordinates** (25 days)	**Understand and apply theorems about circles** **G-C.1** Prove that all circles are similar. **G-C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include[50] the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G-C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **Find arc lengths and areas of sectors of circles**

[50] Relationships include but are not limited to the listed relationships. Example: angles involving tangents and secants (in preparation for Regents Exams).

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
	G-C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector
	Translate between the geometric description and the equation for a conic section
	G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
	Use coordinates to prove simple geometric theorems algebraically
	G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.*
	Apply geometric concepts in modeling situations
	G-MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

Extensions to the Geometry Course

The (+) standards below are included in the Geometry course to provide coherence to the curriculum. They can be used to effectively extend a topic or to introduce a theme/concept that will be fully covered in the Precalculus course. *Note: None of the (+) standard below will be assessed on the Regents Exam or PARRC Assessments until Precalculus.*

Module 2. These standards can be taught as applications of similar triangles and the definitions of the trigonometric ratios.	**Apply trigonometry to general triangles**
	G-SRT.9 (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
	G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

	G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Module 3. This standard on the volume of the sphere is an extension of G-GMD.1. In this course, it is explained by the teacher and used by students in G-GMD.3.	**Explain volume formulas and use them to solve problems** **G-GMD.2** (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
Module 5. This standard is an immediate extension of G-C.2 and can be given as a homework assignment (with an appropriate hint).	**Understand and apply theorems about circles** **G-C.4** (+) Construct a tangent line from a point outside a given circle to the circle.